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THREE DIMENSIONAL ADJUSTMENT AND SIMULATION OF THE EGYPTIAN GEODETIC NETWORK

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ABSTRACT

The Egyptian geodetic network, established 75 years ago, was adjusted in sections by the method of condition equations, in a two dimensional frame. It consists of ten sections with a total of 195 stations. A new adjustment for the ten sections, was done by the author in a three dimensional frame. The effect of using the classical and modern observations in strengthening large geodetic networks is investigated.

1- INTRODUCTION:

In three dimensional geodesy all the observed quantities, horizontal angles or directions, lengths, vertical angles, astronomical latitudes, longitudes, and azimuths are combined in a single adjustment process. Bruns,H.1878, established the basic and fundamental idea of the rigorous computations. Bruns's polyhedron was formed by the triangulation stations on the surface of the earth, the straight lines of sight connecting them and the direction of the plumb line at each station. Accordingly, five parameters at each station, three coordinates and two parameters defining the direction of the plumb line, are needed to determine this figure. A complete derivation of the observation equations used in the adjustment can be found in (Wolf 1963), (Heiskanen & Moritz 1967), and (Shaker,A. 1982) . Generally, at each station the unknowns are differential shifts of position components and of astronomical latitude and longitude. In additions, there are orientation unknowns for horizontal directions, refraction unknowns for vertical angles, and scale unknowns for relative distances.

2- SIMULATION STUDY:

In the absence of a network with all the observational data mentioned before, it was necessary to simulate one for investigating the effect of different types of observations on the standard errors of the station coordinates. The simulation tests were done on two nets, the first in the north-south extension and the second in the east-west extension, both nets were parts of the network of Egypt. The simulated and the real adjustment were carried out

TYPE OF OBSERVATIONS	TYPE OF TESTS	From→To	THE EFFECT ON THE ACCURACY OF STATION COORDINATES				
			GEODETIC			ASTRONOMIC	
			Latitude	Longitude	Height	Latitude	Longitude
DIRECTIONS	CHANGING THE ACCURACY OF OBSERVATIONS. . .	2→0.5" 2→1.0	75% 52%	22% 17%	00% 00%	2% 2%	10% 7%
ASTRONOMICAL AZIMUTH	CHANGING THE NUMBER OF OBSERVATIONS. . .	2 → 20 2 → 8	6% 4%	65% 25%	10% 6%	23% 15%	60% 41%
DISTANCES	CHANGING THE NUMBER OF MEASUREMENTS. . .	2 → 8 2 → 4	62% 45%	00% 00%	00% 00%	00% 00%	00% 00%
VERTICAL ANGLES	CHANGING THE ACCURACY OF OBSERVATIONS. . .	10 → 1 10 → 5	21% 12%	54% 22%	22% 13%	23% 8%	43% 15%
ASTRONOMICAL LATITUDES	CHANGING THE NUMBER OF OBSERVATIONS. . .	2 → 21 2 → 12	10% 2%	21% 7%	80% 55%		28% 9%
ASTRONOMICAL LONGITUDES	CHANGING THE NUMBER OF OBSERVATIONS. . .	2 → 20 2 → 10	2% 2%	53% 23%	2% 2%	27% 21%	

Table (A) NET 1 (NORTH-SOUTH EXTENSION)

TYPE OF OBSERVATIONS	TYPE OF TESTS	From→To	THE EFFECT ON THE ACCURACY OF STATION COORDINATES				
			GEODETIC			ASTRONOMIC	
			Latitude	Longitude	Height	Latitude	Longitude
DIRECTIONS	CHANGING THE ACCURACY OF OBSERVATIONS. . .	2→0.5" 2→1.0	46% 36%	72% 37%	00% 00%	00% 00%	1% 1%
ASTRONOMICAL AZIMUTH	CHANGING THE NUMBER OF OBSERVATIONS. . .	2 →20 2 →10	28% 20%	9% 4%	96% 37%	31% 9%	70% 56%
DISTANCES	CHANGING THE NUMBER OF MEASUREMENTS. . .	2 → 9 2 → 4	00% 00%	57% 52%	00% 00%	00% 00%	00% 00%
VERTICAL ANGLES	CHANGING THE ACCURACY OF OBSERVATIONS. . .	10 → 1 10 → 5	00% 00%	00% 00%	16% 8%	21% 6%	17% 8%
ASTRONOMICAL LATITUDES	CHANGING THE NUMBER OF OBSERVATIONS. . .	2 → 23 2 → 12	4% 2%	7% 7%	5% 3%		21% 9%
ASTRONOMICAL LONGITUDES	CHANGING THE NUMBER OF OBSERVATIONS. . .	2 → 23 2 → 13	21% 17%	6% 2%	90% 81%	30% 6%	

Table (B) NET 2 (EAST-WEST EXTENSION)

in curvilinear coordinates; geodetic latitude, longitude, and ellipsoidal height. Results of the simulation tests are tabulated in tables A, and B.

2-1. Effect of the shape of the net on the standard errors of the adjusted coordinates:

During the simulation study on the north-south and the east-west nets, it was expected that if the s.e. in latitude for the first net is less than that of the longitude, then for the second net the s.e. in longitude will be less than that of latitude. this expectation was due to the fact that both nets have, to a certain extend, the freedom to rotate around the Z-axis of the fixed point, i.e. in the azimuth plane. The results obtained from the simulation were not always in agreement with the above expectation. Accordingly, a theoretical study was done to find out the effect of the shape of the net on the s.e. of the station coordinates.

2-1.1 Error propagation in scale and azimuth:

The models used for the investigation consist of a network with one and two measured azimuths and base lines at the terminals, figure (1). The results of the study gave the following laws for the s.e's in scale and azimuth at P^{th} line along and across the net. For a complete derivation see Shaker, A. 1982.

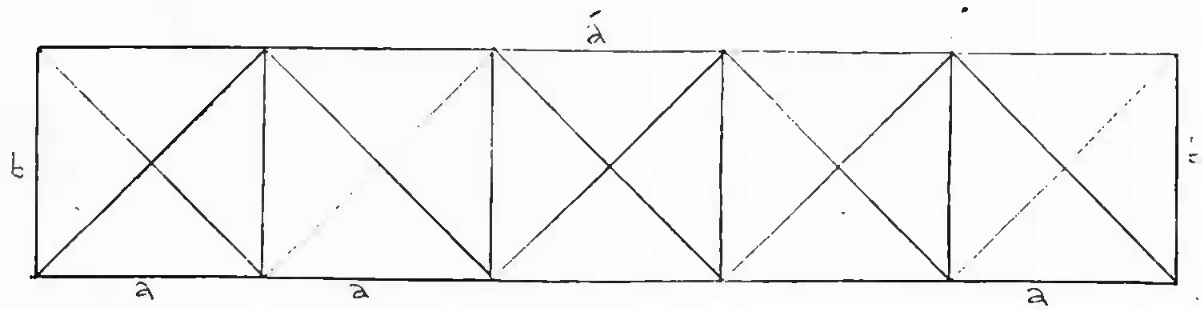

Figure (1)

i) Chain with one measured azimuth and side:

a- Standard error in scale at line P, along the chain

$$\sqrt{M.S^2\left[\frac{5}{8}\ \frac{(1+r^2)}{r} + 0.25 + \frac{(P-1)}{r}\right] + I - \frac{M.S^2(r^2+1)^2}{r.k}}$$

b- Standard error in scale at line P, across the chain

$$\sqrt{P.M.S^2(a^2/b^2)+I}$$

c- Standard error in azimuth at line P, along the chain

$$\sqrt{P.M + N - \frac{M(r-1)^2}{k}}$$

d- Standard error in azimuth at line P, across the chain

$$\sqrt{P.M + N}$$

ii) Chain with two measured azimuths and base lines:

a- Standard error in scale at line P, along the chain

$$= \left[MS^2\left(\frac{5}{8}\cdot\frac{1+r^2}{r} + 0.25 + \frac{(P-1)}{r}\right) + I - \frac{((P-0.5)MS^2+Ir)^2}{r(P_oMS^2+2Ir)} - \frac{MS^2(1+r^2)^2}{rK} - \frac{M^2S^2r}{16(P_oM+2N)}\right]^{1/2}$$

b- Standard error in scale at line P, across the chain

$$= \sqrt{PL + I - \frac{(PL+I)^2}{P_oL + 2I}}$$

c- Standard error in azimuth at line P, along the chain

$$= \left[PM + N - \frac{((P-o.5)M+N)^2}{P_o M+2N} - \frac{(\sqrt{M}-r\sqrt{M})^2}{K} - \right.$$

$$\left. - \frac{M^2 S^2}{4(MS^2+2Ir)} \right]^{1/2}$$

d- Standard error in azimuth at line P, across the chain

$$= \sqrt{PM + N - \frac{(PM + N)^2}{P_o M + 2N}}$$

where,

M....mean value for $(s.e.)^2$ of the measured angles,

N....mean value for $(s.e.)^2$ of the measured azimuths,

I....mean value for $(s.e.)^2$ of the measured base lines,

P_o...Total number of quadrilaterals in the chain,

S.... 10^6 sin 1" & r.... b^2/a^2

k.... $4 + 4r^2$ & L.... $(a/b)^2 MS^2$

The effect of changing the ratio b/a , and b/a' on the s.e. of the coordinates at the end points in a 100 km. chain is given in figure (2). The total length of the chain is denoted by a' and the observations were; horizontal directions (s.e. $\pm$1"), and two measured azimuths (s.e. $\pm$1"), and two base lines (s.e. $\pm$1ppm). Figure (2) shows the best ratio for b/a, which minimizes the difference between the s.e. along and across the net at the end point, lies between 0.5 to 0.9, while the s.e. across and along the chain decreases with the increase of the ratio b/a'.

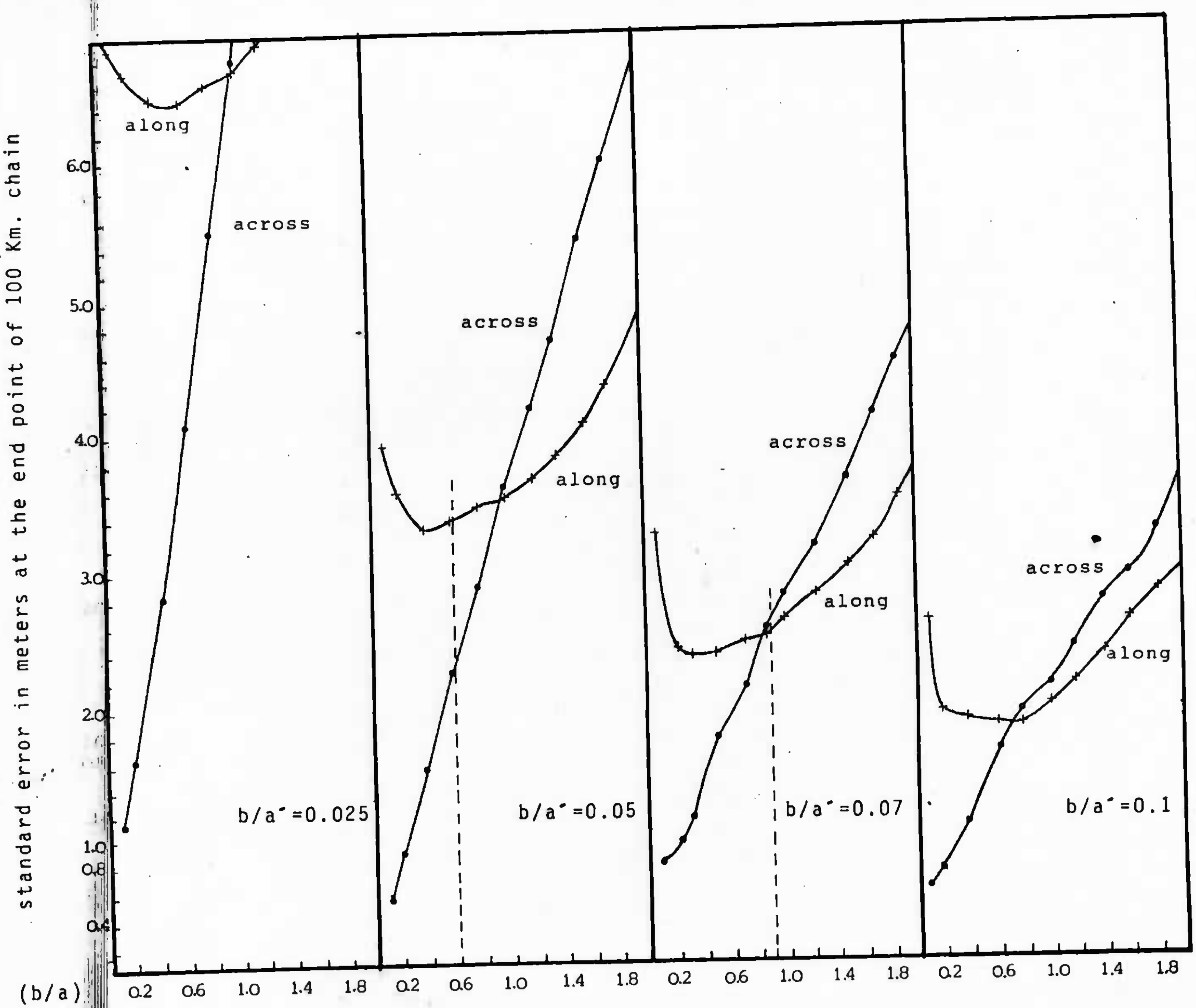
standard error in meters at the end point of 100 Km. chain
(b/a)
along
across
b/a' =0.025
b/a' =0.05
b/a' =0.07
b/a' =0.1
0.2
0.6
1.0
1.4
1.8
0.4
0.8
1.0
2.0
3.0
4.0
5.0
6.0

Fig.(2)

3- ADJUSTMENTS AND RESULTS:

The following three adjustments were done on the Egyptian geodetic network:

i) Adjustment (1): was done on Helmert 1906 ellipsoid with one point held fixed, F1. The geoidal undulations and deflection components at this station were taken as follows

$$N = 0.0 \text{ mt.} \qquad \xi = 3.93'' \qquad \eta = -2.96''$$

$$\phi = 30^{\circ}\ 01'\ 42.86'', \qquad \lambda = 31^{\circ}\ 16'\ 37.028'', \qquad h = 204.64 \text{ mt.}$$

The result shows unequal shifts between the old and the new adjustment, figure (3). The reasons are summarized as follows

a- The azimuth condition was not applied during the old adjustment from Cairo to Aswan (section 1 to 4).

b- Reduction of base lines was based on the heights above the geoid and not on the ellipsoidal heights.

c- The old adjustment was done in sections.

d- No deflection corrections were applied to the observations.

ii) Adjustment (2): was done by incorporating the three available Doppler stations. The results gave the following values for the transformation parameters which will be used to transform from adjustment (2), to the NWL - 9 system.

$$x = -101.48 \text{ mt.}, \qquad R_x = 0.77$$

$$y = 40.14 \text{ mt.}, \qquad R_y = 0.16$$

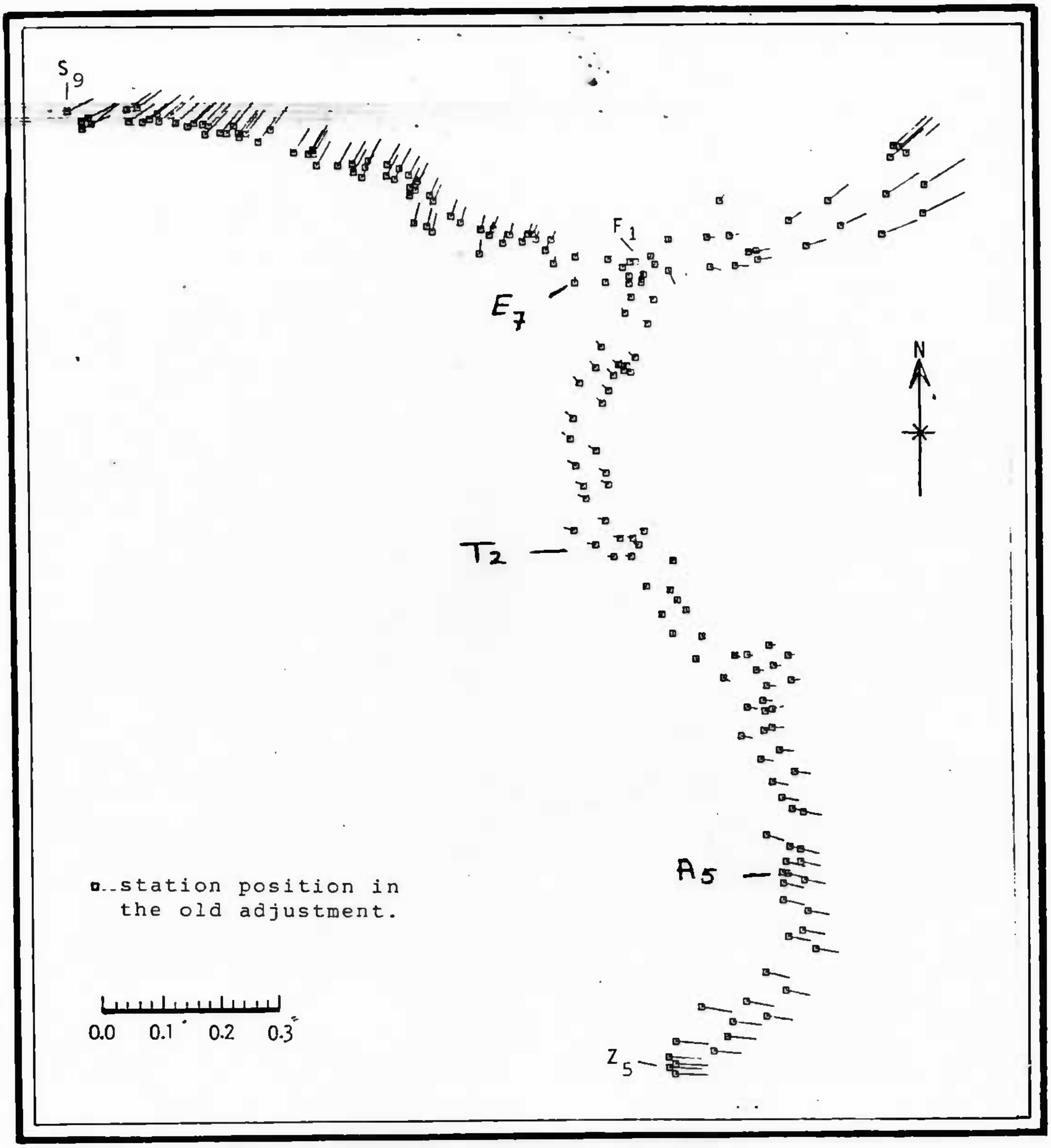

Figure (3)

THE AMOUNT OF SHIFTS IN THE GEODETIC COORDINATES OF THE STATIONS AFTER ADJUSTMENT {1}

$z = -21.36$ mt. , $R_z = 1.74$

Scale Factor = 3.2 ppm.

The effect of introducing the three available Doppler stations, in this adjustment, on stregthening the net was not obvious because the relative accuracies of the lines calculated from the Doppler stations were about the same as that of the corresponding geodetic ones.

iii) Adjustment (3): was done by applying the inner accuracy theory (Meissl,P. 1962,1965,1969), on the geodetic co-ordinate system. The theory allows to filter the effect of an arbitrary set of parameters out of a given covariance matrix. The accuracy remaining after filtering is called inner accuracy. In geodesy the filter parameters, most of the time, are restricted to shifts, rotations, and scale factor. In this case the inner accuracy is that one being liberated from their effect. Most of the geodetic applications of inner accuracy are related to free adjustment. The problem in this case is related to the general case of adjustment of observation equations with constraints. The normal equations with full rank will take the form

$$\begin{bmatrix} N & G \\ G^T & \emptyset \end{bmatrix} \begin{bmatrix} X \\ K_c \end{bmatrix} = \begin{bmatrix} A^T PL \\ \emptyset \end{bmatrix}$$

where

$N_{u.u}$ ... normal equations matrix with rank = u-d,

$G_{u.d}$ ... coefficient matrix of inner constraints with rank = d,

d ... rank deficiency due to lack of datum.

For any point (ϕ , λ) within the net, the corresponding coefficient within the G matrix will take the form;

$$\begin{vmatrix} \cos\phi\cos\phi_0 + \sin\phi\sin\phi_0\cos(\lambda-\lambda_0) & -\sin\phi\sin(\lambda-\lambda_0) & \cos\phi\sin\phi_0 - \sin\phi\cos\phi_0\cos(\lambda-\lambda_0) \\ \sin\phi_0\sin(\lambda-\lambda_0) & \cos(\lambda-\lambda_0) & -\cos\phi_0\sin(\lambda-\lambda_0) \\ \sin\phi\cos\phi_0 - \cos\phi\sin\phi_0\cos(\lambda-\lambda_0) & \cos\phi\sin(\lambda-\lambda_0) & \sin\phi\sin\phi_0 + \cos\phi\cos\phi_0\cos(\lambda-\lambda_0) \end{vmatrix}$$

The value of (ϕ_o , λ_o) is taken as the mean value of (ϕ , λ) in the net..The solution of this system is done by interchanging the inner constraints rows and columns with the last (d.d) rows and columns of the matrix N as follows;

$$\begin{bmatrix} N_{11} & G_1 & N_{12} \\ G_1^T & \emptyset & G_2^T \\ N_{12}^T & G_2 & N_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ K_C \\ X_2 \end{bmatrix} = \begin{bmatrix} A_1^T PL \\ \emptyset \\ A_2^T PL \end{bmatrix}$$

where the dimensions of the above submatrices are

N'_{11} .... (u-d . u-d) , G_1 ... (u-d . d)

N'_{12} .... (u-d . d) , G_2 ... (d . d)

N_{22} .... (d . d)

The free adjustment of the Egyptian net shows that the inner relative accuracies of the lines in the north-south and the east-west extensions are $\simeq$ 1.8, and 2.7 ppm.

respectively. By introducing the Doppler stations, for example, at station E_7 and A_5, in the north-south extension we achieve a relative accuracy of $\simeq$ 1.8 ppm. which is the same value as obtained for the Egyptian net. If this separation was less than the existing one, then the Egyptian net will be superior in the accuracy, and the Doppler stations will be of no use except the positioning of the Egyptian datum with respect to the geocenter of the earth. The minimum effective separation between the Doppler stations which fits the accuracy of the net of Egypt is given in table (c).

Doppler accuracy	The Separation in Km.	
	N-S extension	E-W extension
± 0.5 mt.	392	262
± 1.0 mt.	784	524

Table (c)

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